

8.4 – Matrices for General Linear Transformations

Recall this example from section 1.8

Find the standard matrix A for the linear transformation $T : R^2 \rightarrow R^3$ for which

$$T \left(\begin{bmatrix} -1 \\ 2 \end{bmatrix} \right) = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \text{ and } T \left(\begin{bmatrix} 3 \\ -5 \end{bmatrix} \right) = \begin{bmatrix} 5 \\ -7 \\ 1 \end{bmatrix} \text{ and use it to compute } T \left(\begin{bmatrix} -4 \\ 3 \end{bmatrix} \right).$$

And #3 from 4.7

Consider the bases $B = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ and $B' = \{\mathbf{u}'_1, \mathbf{u}'_2, \mathbf{u}'_3\}$ for R^3 , where

$$\mathbf{u}_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \mathbf{u}_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \text{ and } \mathbf{u}'_1 = \begin{bmatrix} 3 \\ 1 \\ -5 \end{bmatrix}, \mathbf{u}'_2 = \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}, \mathbf{u}'_3 = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

a. Find the transition matrix B to B' .

b. Compute the coordinate vector $[\mathbf{w}]_B$, where $\mathbf{w} = \begin{bmatrix} -5 \\ 8 \\ -5 \end{bmatrix}$ and use the transition matrix in part (a) to compute $[\mathbf{w}]_{B'}$.

Now we combine these two concepts.

#5 Let $T : R^2 \rightarrow R^3$ be defined by $T \left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 + 2x_2 \\ -x_1 \\ 0 \end{bmatrix}$.

a. Find the matrix $[T]_{B',B}$ relative to the bases $B = \{\mathbf{u}_1, \mathbf{u}_2\}$ and $B' = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, where

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}, \mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$





